Assessment of the polynomials for conversion between Geodetic coordinates (φ, λ) and the UTM "Or Vice Versa"

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مع استقرار تقنيات الجيوديسيا الفضائية "وخاصة نظام الـ GPS" كمصدر رئيسى للبيانات المساحية المستخدمة فى إنتاج الخرائط ، إنتشر تحديد موضع النقاط بإستخدام الإحداثيات المنسوبة للنظام الجيوديسى العالمى 1984 واسقاطها بإستخدام "نظام إسقاط مركيتور المستعرض العالمى". ونظرا لزيادة التعامل مع الخرائط وظهور بعض مستخديمها من غير المتخصصين فى علوم المساحة وغير الملمين بقواعد التحويل بين نظم الاحداثيات المختلفة ومبادئ علم إسقاط الخرائط ، تم تقديم هذا البحث والذى يقوم على فكرة تحويل الإحداثيات الجيوديسية (φ, λ) وإسقاطها على "نظام إسقاط مركيتور المستعرض العالمى UTM " و كذلك العملية العكسية وهى الحصول على الإحداثيات الجيوديسية المقابلة لإحداثيات النقاط فى UTM . وذلك بإستخدام كثيرات الحدود ذات الدرجات المختلفة. حيث تم تقسيم مصر إلى خمس مناطق طولية وتم تحديد درجة كثيرات الحدود المناسبة بإستخدام نقاط إختبار عند كل منطقة. والتى أعطت نتائج دقيقة فى الدرجات الرابعة والخامسة طبقاً للمنطقة.

<u>Abstract</u>

Maps, either in "traditional or digital forms" are considered the prime source for spatial data that applied to represent the shape of the earth and to define the features positions.

According to the recent advancement of the satellite geodesy, especially GPS, as a major source of the surveying data that applied for mapping purposes and produces coordinates that defined with respect to the World Geodetic System 1984.

The coordinates of GPS are taken directly everywhere and projected for mapping using UTM projection. Most of the users are not familiar with the details of coordinate transformation and projections. Therefore, this paper is prepared to help the non specialized user to exchange between geodetic coordinates (φ, λ) to UTM projection and vice versa using the polynomial with different orders. The area of Egypt (25° E to 37° E) is divided into 5 regions. The behavior of the polynomial is assessed using check points to specify the proper order with each region. The 4th and 5th order gave the best results according to the particular region. In order to test the proposed methodology a computer package is designed and the results are analyzed and checked.

Introduction

Nowadays, the global positioning system is considered the primary source of providing surveyors with data, where it provides new opportunities for surveyors and their professions.

The GPS has the potential to revolutionize the practice of surveyors, to provide new meaning to the numerical records of surveyors, and to make the results "coordinates and maps" available to others from related disciplines [Lieck 2000]. GPS provides us with the geodetic coordinates (φ, λ) with respect to the WGS84 datum, and then we project the resulted date into the Universal Transverse Mercator "UTM". The UTM is one of the several grids that have been devised to simplify the problems of giving directions, distances and positions.

The UTM is generated using a uniform international system of longitude zones, where there are 60 zones, each 6 degrees of longitude in width Fig (1). Zone numbers begin at the International Dateline (180 degrees longitude) and are numbered east around the world from 1 to 60. Thus zone 30 and zone 31 are on the west and east sides of the Greenwich meridian respectively. Each of these zones has a Central Meridian (CM) equivalent to the longitude at the center of the zone. Thus a coordinate transformation for any location within a specific UTM zone will use the CM defined by this zone system [McDonnell 1979]. The UTM projection is limited in its east – west extend by the maximum allowable scale distortion, and is limited in northern hemisphere to 80º latitude and the southern hemisphere to 80º latitude "the 60 UTM and the two Universal Polar Stereographic "UPS" are covering the whole world". The unit of measure is the meter and the scale factor at central meridian is 0.9996. False northing is 0.0 (for the southern hemisphere, a false northing of 10,000,000 m is used), while false easting of 500,000 m is used for the central meridian of each zone. The reference ellipsoid used is WGS84, where zones 35, 36 and 37 are covering the area of Egypt [Strahler, 1987].

Any point can then be described by its distance east of the origin (its 'easting' value).

By definition the Central Meridian is assigned a false easting of 500,000 meters. Any easting value greater than 500,000 meters indicates a point east of the central meridian. Any easting value less than 500,000 meters indicates a point west of the central meridian. Distances (and locations) in the UTM system are measured in meters, and each UTM zone has its own origin for east-west measurements [Steven, 2003].

To eliminate the necessity of using negative numbers to describe a location, the east-west origin is placed 500,000 meters west of the central meridian. This is referred to as the zone's 'false origin'. The zone doesn't extend all the way to the false origin.

The origin for north-south values depends on whether you are in the northern or southern hemisphere. In the northern hemisphere, the origin is the equator and all distances north (or 'nothings') are measured from the equator [Seeper, 2003].

1. Converting Latitude and Longitude $(\varphi, \lambda)_{WGS84}$ to UTM grid coordinates

When using GPS, the coordinates of the terrestrial sites are obtained in the same reference frame. Since the WGS84 is a geocentric system, so the objective is to map any point (φ, λ) on the ellipsoid into the equivalent (E,N) on the plan.

The origin of the map "E-coordinate" is the equator, where the properties of this projection are:

(1) Both the central meridian and the normal to it are represented by straight lines.

(2) Other meridians are complex curves that are concave towards the central meridian,

(3) Parallel are concave curves towards the pole and

(4) The scale is true only along the central meridian.

The projection formulas [Mikhail, 1998] are:

$$E = Nr \left[(\lambda^r) \cos \varphi + ((\lambda^r)^3 \cos^3 \varphi /6)(1 - \tau^2 + \eta^2) + ((\lambda^r)^5 \cos^5 \varphi / 120) \right. \qquad \ldots\ldots\ldots\ldots\ldots(1)$$
$$\left. (5 - 18 \tau^2 + \tau^4 + 14 \eta^2 - 58 \tau^2 \eta^2 + 13 \eta^{4)} + \ldots\ldots\ldots) \right]$$

While

$$N = MA + Nr \left[((\lambda^r)^2/2)(\sin \varphi \cos \varphi) + ((\lambda^r)^4 / 24) \sin \varphi \cos^3 \varphi)(5 - \tau^2 \right.$$
$$+ 9 \eta^2 + 4 \eta^4) + ((\lambda^r)^6 / 720) \sin \varphi \cos^5 \varphi \ (61 - 58 \tau^2 + \tau^4 + 270 \eta^2 \qquad \ldots\ldots\ldots\ldots\ldots(2)$$
$$\left. - 330 \tau^2 \eta^2 + 445 \eta^{4)} + \ldots\ldots\ldots) \right]$$

Where:

$Nr = a / (1 - \eta^2 \varepsilon^2 \sin^2 \varphi)^{0.5}$

$\tau = \tan \varphi$

$\eta^2 = \varepsilon^2 \cos^2 \varphi /(1 - \varepsilon^2) = ((a^2 - b^2) \cos^2 \varphi)/b^2$

$MA = {}_0\!\int^\varphi Mr \, d\varphi = {}_0\!\int^\varphi (a(1 - \varepsilon^2)/(1 - \varepsilon^2 \sin^2 \varphi)) \, d\varphi$

$MA = a(A_o \varphi^r - A_1 \sin 2\varphi + A_2 \sin 4\varphi - A_3 \sin 6\varphi + \ldots\ldots)$

In which

$A_o = 1 - (1/4) \varepsilon^2 - (3/64) \varepsilon^4 - (5/256) \varepsilon^6 - \ldots\ldots$

$A_1 = (3/8) \varepsilon^2 + (3/32) \varepsilon^4 + (45/1024) \varepsilon^6 + \ldots\ldots$

$A_2 = (15/256) \varepsilon^4 - (45/1024) \varepsilon^6 + \ldots\ldots$

$A_3 = (35/3072) \varepsilon^6 + \ldots\ldots$

a is the semi major axis of the used ellipsoid

ε is the first eccentricity of the used ellipsoid

φ is the geodetic latitude of the point

φ^r is the geodetic latitude of the point in radian

λ^r is the geodetic longitude of the point in radian

2. Converting UTM to Latitude and Longitude

It is highly recommended to perform the inverse mapping procedure (from the UTM to the geodetic coordinates) to demonstrate the achievable accuracy. Relative to the central meridian; subtract 500,000 from conventional UTM coordinate.

Calculate the Meridian Arc (MA)

$MA = Nr/k_0$.

Where k_0 is the scale factor at the central meridian = 0.9996

Calculate Footprint Latitude (fp)

$$mu = MA/[a(1 - \varepsilon 2/4 - 3 \varepsilon 4/64 - 5 \varepsilon 6/256...) \qquad \ldots\ldots\ldots\ldots(3)$$
$$e1 = [1 - (1 - \varepsilon 2)1/2]/[1 + (1 - \varepsilon 2)1/2] \qquad \ldots\ldots\ldots\ldots(4)$$
$$\text{footprint latitude } fp = mu + J1\sin(2mu) + J2\sin(4mu) + J3\sin(6mu) + J4\sin(8mu) \qquad \ldots\ldots\ldots\ldots (5)$$

where:

$J1 = (3e_1/2 - 27e_1^3/32 ..)$

$J2 = (21e_1^2/16 - 55e_1^4/32 ..)$

$J3 = (151e_1^3/96 ..)$
$J4 = (1097e_1^4/512 ..)$

Calculate Latitude and Longitude

$e'^2 = (\varepsilon \, a/b)^2 = \varepsilon^2/(1- \varepsilon^2)$
$C1 = e'^2\cos^2(fp)$
$T1 = \tan^2(fp)$
$R1 = a(1-e^2)/(1-e^2\sin^2(fp))^{3/2}.$
$N_r = a/(1-e^2\sin^2(\varphi))^{1/2}.$
$D = E/(N_r k_0)$

$\varphi = \underline{fp} - Q1\,(Q2 - Q3 + Q4),$ ………….………(6)

 Where:

$Q1 = N_r \, \tan(fp)/R1$
$Q2 = (D^2/2)$
$Q3 = (5 + 3T1 + 10C1 - 4C1^2 - 9e'^2)D^4/24$
$Q4 = (61 + 90T1 + 298C1 + 45T1^2 - 3C1^2 - 252e'^2)D^6/720$

$\lambda = \lambda_0 + (Q5 - Q6 + Q7)/\cos(\underline{fp})$ ……….………(7)

Where:

- $Q5 = D$
- $Q6 = (1 + 2T1 + C1)D^3/6$
- $Q7 = (5 - 2C1 + 28T1 - 3C1^2 + 8e'^2 + 24T1^2)D^5/120$

The presented Methodology

Another alternative to the conversion equations can be carried out using Polynomial transformations between the two coordinate systems, where the small distortions are then approximated by polynomial functions in easting and northing.

Depending on the degree of variability in the distortion, approximation may be carried out using 3rd, 4th or higher degree polynomials. In the case of projection between two coordinate systems, the additional distortions resulting from the application of two map projections and a datum transformation can be included in a single polynomial approximation functions that are subjected to variations, as different approximation characteristics may be achieved by different polynomial functions.

The simplest of all polynomials is the general polynomial function [Anderson and Mikhail, 1998]:

$E = a_o + a_1 \varphi + a_2\lambda + a_3 \varphi^2 + a_4 \varphi \lambda + a_5 \lambda^2$ ……….…… (2nd order)
$\quad + a_6 \varphi^3 + a_7 \varphi^2 \lambda + a_8 \varphi \lambda^2 + a_9 \lambda^3$ …................. (3rd order)
$\quad + a_{10} \varphi^4 + a_{11} \varphi^3 \lambda + a_{12} \varphi^2 \lambda^2 + a_{13} \varphi \lambda^3 + a_{14} \lambda^4$ …………..…… (4th order)
$\quad + a_{15} \varphi^5 + a_{16} \varphi^4 \lambda + a_{17} \varphi^3 \lambda^2 + a_{18} \varphi^2 \lambda^3$ ………………….. (5th order)
$\quad + a_{19} \varphi \lambda^4 + a_{20} \lambda^5$

………………………(8)

$N = b_o + b_1 \varphi + b_2\lambda + b_3 \varphi^2 + b_4 \varphi \lambda + b_5 \lambda^2$ ……….…… (2nd order)
$\quad + b_6 \varphi^3 + b_7 \varphi^2 \lambda + b_8 \varphi \lambda^2 + b_9 \lambda^3$ …................. (3rd order)
$\quad + b_{10} \varphi^4 + b_{11} \varphi^3 \lambda + b_{12} \varphi^2 \lambda^2 + b_{13} \varphi \lambda^3 + b_{14} \lambda^4$ …………..…… (4th order)

$$+ \ b_{15} \ \varphi^5 + b_{16} \ \varphi^4 \lambda \ + b_{17} \ \varphi^3 \lambda^2 + b_{18} \ \varphi \ \lambda^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5^{th} \ order)$$
$$+ b_{19} \ \varphi \ \lambda^4 + b_{20} \lambda^5$$

…………………….(9)

The number of polynomial terms can be calculated from the equation:

…………………………..…….(10)

$$P = (\frac{n^2 + 3n}{2}) + 1$$

Where: n   is the degree of the polynomial.
              P   is the number of polynomial coefficients.

Data Selection, algorithm and Processing

Testing the proposal is done using the data set which can be described as follows:
Egypt includes zones 35, 36 and 37. For zone 35, it covers the area from 25^0 E till 30^0 E, while zone 36 includes the area from 30^0 E till 36^0 E. For zone 37 is covering only one degree from 36^0 E till 37^0 E. For the computations in this article, the following are made:

- Every zone is divided into 15′ x 15′ grids. The geodetic coordinates of the grid points are joined to WGS84 values $(\varphi, \lambda)_{WGS84}$.
- These geodetic coordinates are projected using UTM projection and $(E, N)_{UTM}$ are obtained.
- After all, two files are generated. The first one contains UTM coordinates for the grid points and the second one contains $(\varphi, \lambda)_{WGS84}$ coordinates for the same points.
- Through these files, the polynomial coefficients are computed, and
- The interpolated output "either (φ, λ) or UTM" are obtained.
Fig (3) represents the processes that were followed to design the applied software package.

An example is the polynomial approximation between (φ, λ) and UTM in zone1, the output of the polynomial functions have corrections to the input coordinates within a few centimeters. These small differences prove the property that makes a polynomial function reversible in practice.
The area of Egypt (25 E to 37 E) is divided to the following regions for the easiness of computations:

- Region 1 $(\varphi, \lambda)_{WGS84}$. from 25º E to 27º E exists in Zone 35 UTM
- Region 2 $(\varphi, \lambda)_{WGS84}$. from 27º E to 30º E exists in Zone 35 UTM
- Region 3 $(\varphi, \lambda)_{WGS84}$. from 30º E to 33º E exists in Zone 36 UTM
- Region 4 $(\varphi, \lambda)_{WGS84}$. from 33º E to 36º E exists in Zone 36 UTM
- Region 5 $(\varphi, \lambda)_{WGS84}$. from 36º E to 37º E exists in Zone 37 UTM. See Fig (2)

For each region, the polynomial coefficients are calculated, where the same polynomial coefficients are used to execute both the direct and the reverse process especially when they are applied to the approximation of small differences between a set of points in two different coordinate systems.

Results

After dividing the whole area into grids 15′ x 15′ in the 5 regions, polynomials of 3rd, 4th, 5th and 6th orders are used in relating (φ, λ) to UTM coordinates and vice versa. Every time, the coefficients of the polynomials are computed and check points are used to assess the quality of the applied polynomials. Ten check points in every region are used; the positions of the points covered every region. The results of the computations were represented in the following tables and figures as follow:

Table 1: Residuals of the check points in region 1 obtained from the different orders polynomials
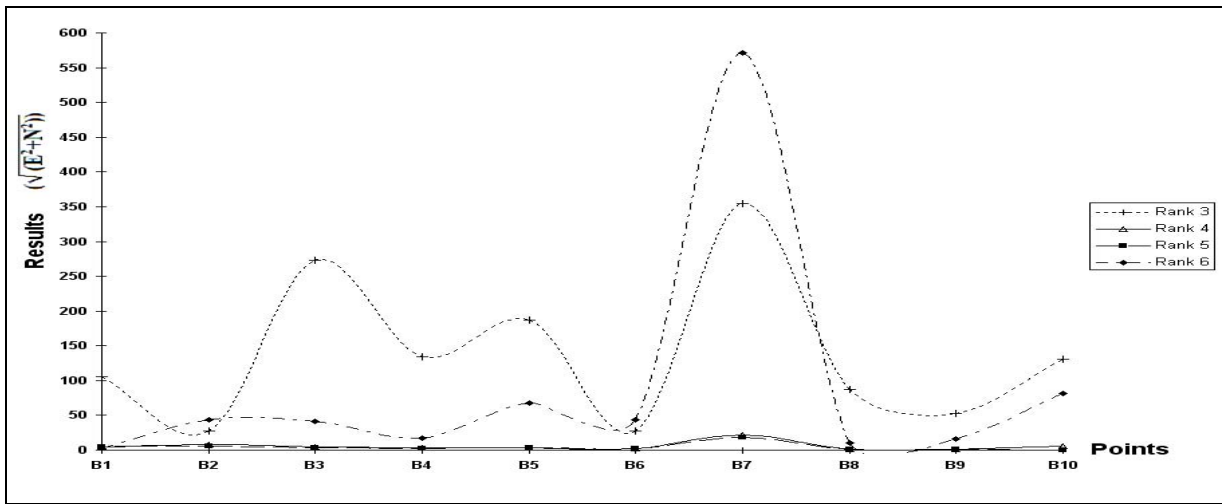
	Residuals in cm							
	3rd order		4th order		5th order		6th order	
Pts	E	N	E	N	E	N	E	N
A1	-17.64	95.70	0.12	3.83	0.07	-2.14	10.45	7.02
A2	1.38	-21.33	-0.11	0.93	0.53	1.23	-4.81	-9.31
A3	-51.94	-18.94	-2.17	-3.38	-0.52	-3.48	-4.04	5.66
A4	28.28	-7.17	-0.97	-3.89	-0.37	1.05	-1.25	8.56
A5	5.61	-59.91	0.63	0.08	-0.04	-0.44	7.93	9.34
A6	-16.27	108.26	2.43	2.24	0.31	1.18	3.45	7.07
A7	-0.41	-56.15	0.28	5.89	0.11	-3.30	-6.26	-4.46
A8	15.44	65.77	0.37	0.97	0.20	-1.12	7.97	18.54
A9	3.84	28.21	0.09	5.05	0.14	-3.23	-3.10	4.84
A10	-41.28	-9.05	-0.50	-4.13	-0.45	2.50	10.48	-3.05
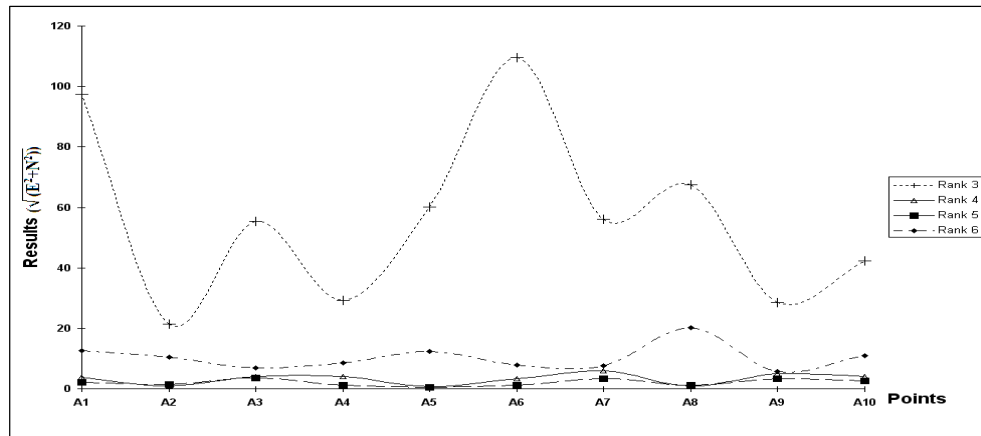
Looking at the residuals in Table (1), all the different orders have undersized residuals "regardless of the 3rd order one". Herein, the 5th orders provide relatively better values.

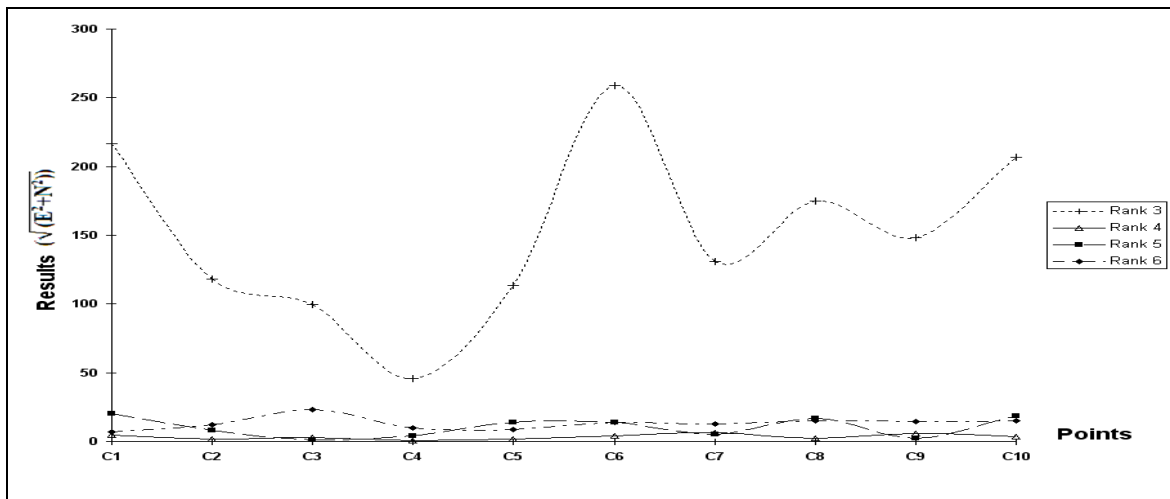Table 2: Residuals at the check points in region 2 from the different orders

	Residuals in cm							
	3rd order		4th order		5th order		6th order	
Pts	E	N	E	N	E	N	E	N
B1	-82.97	65.76	-0.24	4.37	-0.73	4.24	2.32	-0.68
B2	-1.16	-27.23	0.55	-8.13	-0.32	-5.68	26.34	35.22
B3	-177.57	208.16	-0.75	-5.08	1.64	-2.91	-27.26	-31.60
B4	8.27	-133.51	-0.20	-3.29	0.44	-1.69	-8.89	-15.24
B5	70.46	173.01	3.06	-2.38	0.11	-3.19	-46.97	-48.87
B6	18.54	-21.07	-1.32	2.09	0.24	2.59	-32.94	-28.03
B7	-84.26	-344.72	-2.53	-22.00	-1.33	-17.86	385.44	421.73
B8	23.84	83.36	0.48	-0.88	-0.59	0.10	-8.87	-4.86
B9	-26.49	-45.84	0.22	1.37	0.63	1.46	-11.56	-11.52
B10	128.13	-23.64	-2.59	-5.62	-0.13	-0.35	58.53	56.65

In region 2, the residuals are decreasing with increasing the order of the used polynomial until the 5th order then they increase again in the 6th order. The 5th order polynomial gave the best values, almost few centimeters.

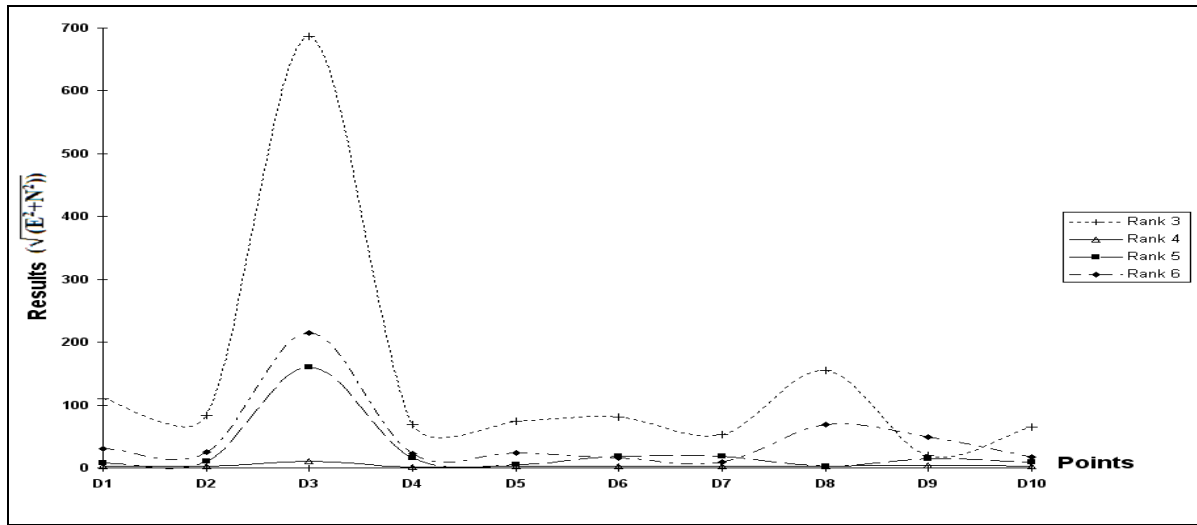Table 3: Residuals at the check points in region 3 from the different orders

| | Residuals in cm | | | | | | | |
| | 3rd order | | 4th order | | 5th order | | 6th order | |
Pts	E	N	E	N	E	N	E	N
C1	133.68	-170.34	4.08	-2.34	-20.22	-1.31	-5.61	-4.08
C2	-117.50	-14.54	-1.69	-0.43	-7.95	-0.22	8.63	-8.65
C3	-2.45	-99.40	-0.67	-2.57	-1.00	-0.57	-22.37	-4.85
C4	45.49	-0.88	-0.40	0.75	-4.00	-0.28	10.05	0.35
C5	-112.71	-14.56	-1.50	0.45	-14.07	-0.82	7.69	3.55
C6	258.31	13.81	0.27	4.09	14.15	0.87	-3.45	13.47
C7	-130.89	-3.91	6.26	0.92	5.31	0.26	-11.84	-5.01
C8	160.77	68.83	-2.46	-0.67	-16.61	-1.20	6.86	13.18
C9	71.56	129.62	-5.43	-2.37	-2.09	-0.01	9.63	-11.15
C10	-183.14	95.37	1.08	-3.56	-18.27	-1.69	-15.15	0.29

Similar results to the previous region are obtained. The 4th order provides the best results.

Table 4: Residuals at the check points in region 4 from the different orders

| Pts | Residuals in cm | | | | | | | |
| | 3 rd order | | 4 th order | | 5 th order | | 6 th order | |
	E	N	E	N	E	N	E	N
D1	-75.05	-80.40	-2.61	0.49	7.08	-4.53	-26.75	-13.54
D2	80.56	23.55	-2.03	-0.47	-9.62	2.91	-25.19	1.88
D3	-679.93	95.88	10.41	2.22	155.86	-39.71	213.64	26.12
D4	-65.77	18.38	-0.72	0.62	15.23	-2.80	-22.06	5.20
D5	37.11	-64.90	-2.95	0.88	5.63	-1.96	22.92	-4.46
D6	-78.45	-18.06	-1.92	0.84	17.02	-5.57	15.89	-4.67
D7	-46.31	-26.33	2.85	0.22	-18.13	5.05	9.63	0.92
D8	-152.90	23.91	-1.67	-2.68	-1.38	-1.74	67.16	-12.83
D9	18.17	-7.06	4.22	-0.01	-13.77	2.59	-48.76	-0.60
D10	-58.68	28.33	-2.07	-0.59	-8.79	3.92	-16.74	0.63

Recalling that, the width of region 2, 3 and 4 is 3˚ while the width of region 1 is 2˚. Herein, the 4th order produces the best results.

Table 5: Residuals at the check points in region 5 from the different orders

| Pts | Residuals in cm | | | | | | | |
| | 3 rd order | | 4 th order | | 5 th order | | 6 th order | |
	E	N	E	N	E	N	E	N
E1	-13.54	17.30	-1.24	0.68	1.05	-0.38	-13.63	5.36
E2	9.04	37.45	1.98	0.54	-2.57	0.02	6.85	-7.52
E3	-19.36	7.34	0.51	0.49	-2.93	-0.20	4.28	-2.66
E4	-12.36	-17.54	-3.54	0.59	1.74	-0.38	-10.18	5.79
E5	-9.06	-29.43	4.57	-0.62	-3.91	0.40	13.58	-5.96
E6	-5.66	-48.90	8.57	1.20	-3.89	-0.44	-4.66	2.30
E7	19.26	7.02	-6.30	-1.47	2.87	0.70	-1.17	4.52
E8	-15.59	8.90	1.85	0.02	-0.46	0.04	-6.27	2.38
E9	-10.25	-9.27	0.15	-0.62	1.28	0.41	1.91	5.92
E10	-20.64	-10.51	-1.42	-0.61	0.14	0.15	0.41	10.08
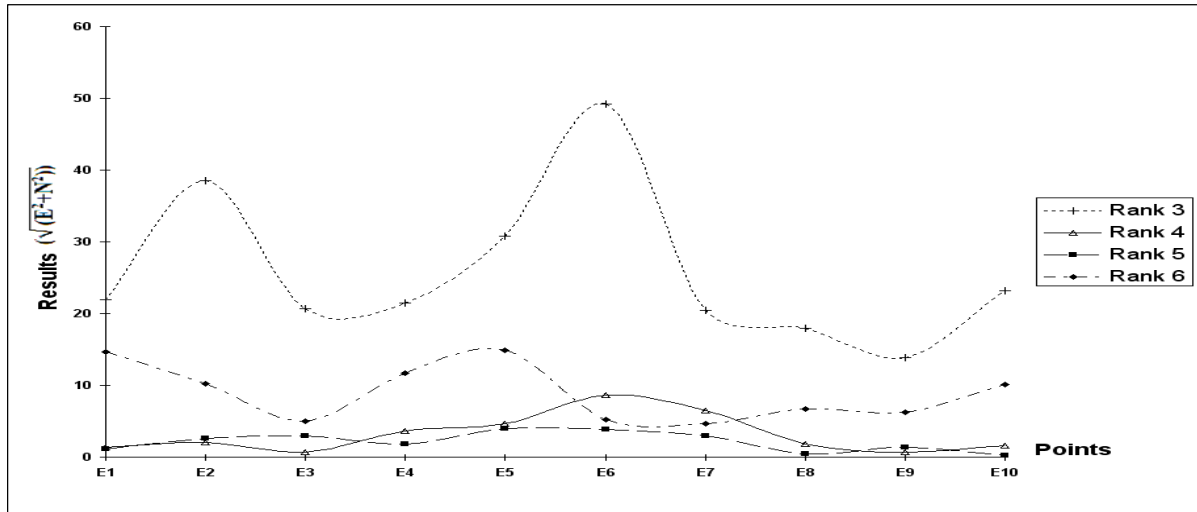
In region 5 "1˚ width", the 5th order polynomial gave the best results.

After illustrating all the solutions over all the examined regions, it can be concluded that the fifth order polynomial in the regions (1,2,5) and the fourth order polynomial in the regions (3,4) gave the best results.

On the other hand, converting the UTM grids to the geodetic coordinates (φ, λ) can be implemented using the same polynomial coefficients.

Recalling the accessible polynomial coefficients, we started to convert any given UTM grids into the equivalent geodetic coordinates (φ, λ).

It should mention here that, the obtainable residuals are converted from seconds to centimeters "for better understanding to the output values".

The following tables represent the same procedures that followed in the previous section.

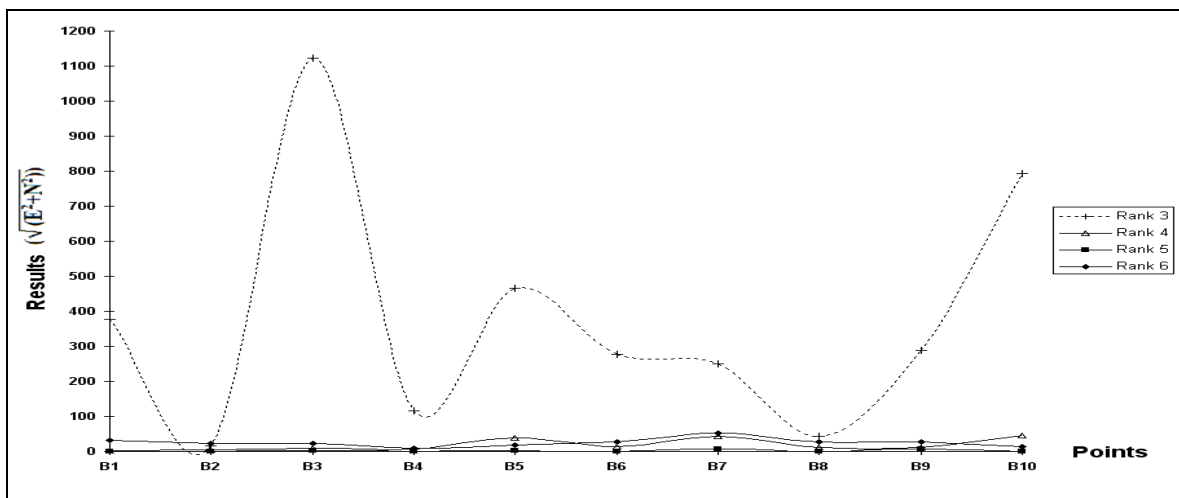Table 6: Residuals at the check points in region 1 from the different orders

	Residuals in cm							
	3 rd order		4 th order		5th order		6th order	
Pts	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.
A1	59.30	-81.40	2.38	0.03	1.00	0.61	-2.77	0.36
A2	-13.96	11.28	-1.90	-0.06	0.77	-0.44	5.95	-6.38
A3	-21.90	-386.27	2.26	-33.12	2.39	0.19	-5.68	-9.80
A4	2.16	232.51	-0.23	-14.11	-0.39	-0.80	-3.03	6.24
A5	-28.44	42.65	0.24	10.69	-0.01	-0.29	-6.20	6.22
A6	70.92	-130.52	0.76	34.10	-1.05	0.09	-1.58	-0.82
A7	-34.73	-8.77	2.18	0.28	0.07	0.75	2.07	-7.56
A8	32.26	41.39	0.08	4.46	-0.28	-0.85	-2.65	9.77
A9	5.74	41.31	1.95	-2.62	-0.58	-0.21	4.03	-5.62
A10	-9.10	-213.02	0.50	-6.94	-0.25	0.37	-5.78	2.76
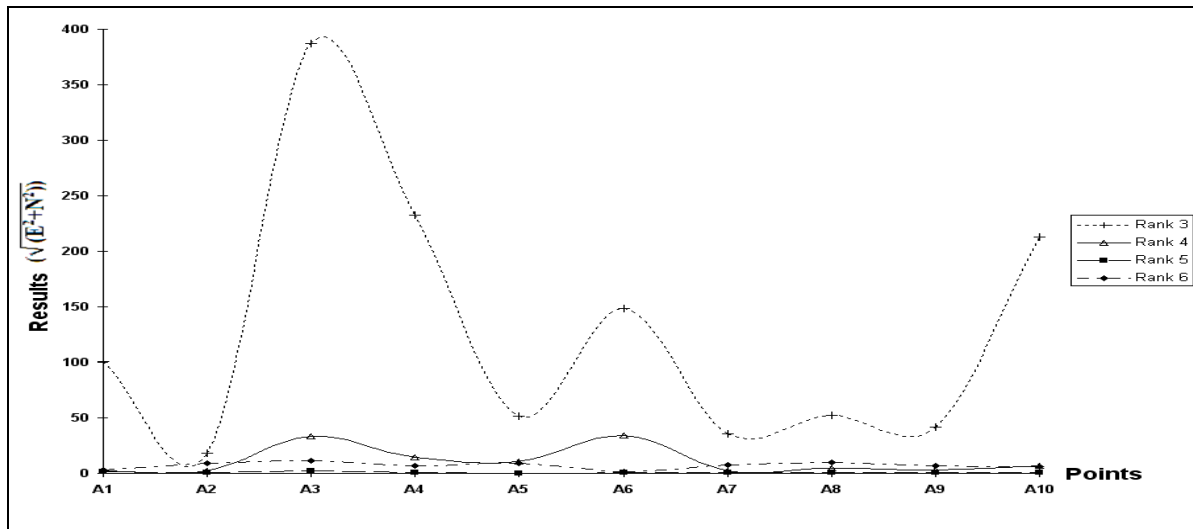
The same behavior for this region to convert from UTM to φ, λ, also, the order that give the best value is the 5th order polynomial.

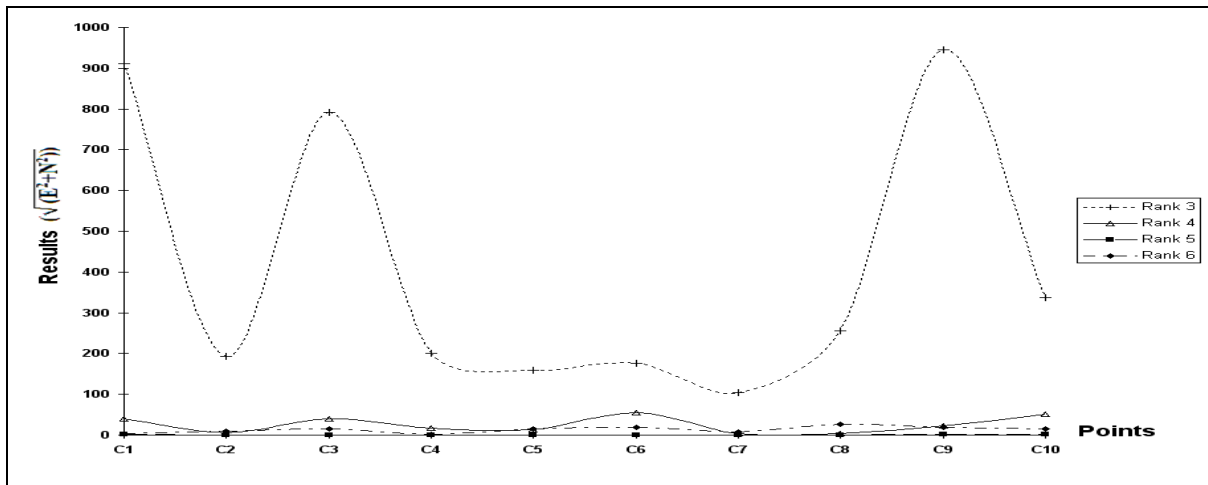Table 7: Residuals at the check points in region 2 from the different orders

| | Residuals in cm | | | | | | | |
| | 3 rd order | | 4 th order | | 5 th order | | 6 th order | |
Pts	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.
B1	41.76	-374.59	0.71	-2.28	0.12	0.55	-30.38	-10.78
B2	-16.51	0.73	-4.69	0.61	0.40	-0.38	20.90	6.27
B3	121.87	-1115.88	9.93	1.69	0.06	2.46	19.44	13.60
B4	-71.37	92.50	-1.46	-6.52	-0.26	0.56	-7.92	-6.08
B5	108.58	453.89	-2.74	38.87	-0.01	-1.34	-15.84	-8.90
B6	-21.31	276.55	3.53	-13.60	-0.09	0.31	26.05	8.75
B7	-214.29	130.08	-2.21	-42.14	-1.25	-6.14	-51.71	-2.98
B8	37.56	-22.87	0.72	12.20	-0.50	-0.96	-25.87	-9.25
B9	-31.80	-285.86	0.55	10.26	-6.49	0.51	26.63	8.18
B10	-20.29	793.66	-5.14	-46.02	0.01	0.71	12.92	5.83

In region 2, the residuals are decreasing with increasing the order of the used polynomial until the 5th order then they increase again in the 6th order. The 5th order polynomial gave the best values, almost few centimeters.

Table 8: Residuals at the check points in region 3 from the different orders

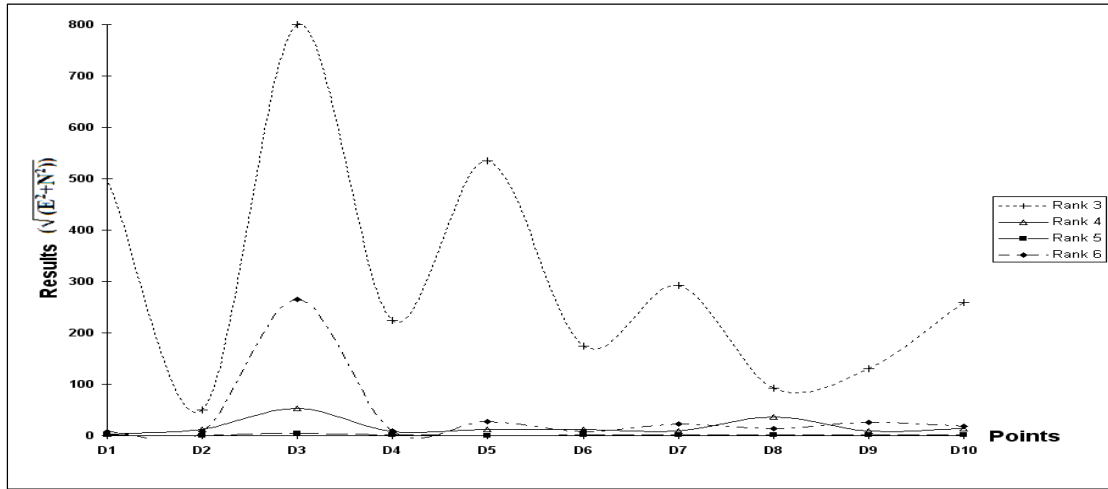| | Residuals in cm | | | | | | | |
| | 3rd order | | 4th order | | 5th order | | 6th order | |
Pts	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.
C1	-85.62	-906.01	-4.26	-40.06	-0.40	-1.26	-0.73	3.29
C2	65.02	-181.33	4.86	-1.67	-0.40	0.51	-4.96	8.88
C3	16.62	-791.67	-2.41	-39.68	-0.06	-0.59	-6.59	14.68
C4	-23.21	200.05	2.61	17.21	0.44	0.57	-1.84	2.11
C5	60.34	-147.63	1.26	12.58	-0.33	0.50	6.32	-14.06
C6	-162.01	67.32	-0.02	55.29	0.39	-0.09	7.92	-16.61
C7	77.70	-69.14	-4.63	0.30	0.38	-0.32	-3.56	7.61
C8	-93.79	237.32	2.43	-2.77	-0.33	0.02	11.77	-23.40
C9	-45.30	944.73	4.83	-22.54	0.11	-1.14	-7.12	18.03
C10	99.52	322.80	2.93	-51.81	-0.82	2.29	4.34	-15.04

Similar results to the previous region are obtained. The 5th order still the best "still few centimeters".

Table 9: Residuals at the check points in region 4 from the different orders

| | Residuals in cm | | | | | | | |
| | 3rd order | | 4th order | | 5th order | | 6th order | |
Pts	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.	LAT.	LONG.
D1	46.29	-497.36	1.95	2.41	0.25	1.84	-6.52	3.37
D2	-43.81	-23.11	0.34	-12.67	0.36	0.49	1.07	-7.35
D3	399.63	1727.43	18.70	50.35	-4.63	0.27	-40.77	-261.56
D4	38.33	221.58	4.37	6.13	0.01	-0.98	4.72	7.45
D5	-25.26	-534.59	-0.95	11.38	-0.16	-0.07	-18.06	-20.97
D6	38.87	-169.48	1.29	12.75	-0.80	-0.23	1.29	-7.65
D7	33.05	-290.51	-3.81	7.44	0.84	0.41	-7.77	-21.78
D8	92.30	11.44	-0.02	-35.93	-0.35	1.41	-12.87	3.57
D9	0.90	-130.31	-3.81	8.01	0.48	-0.89	-15.50	-20.46

	Residuals in cm							
	3 order		**4 order**		**5 order**		**6 order**	
Pts	**LAT.**	**LONG.**	**LAT.**	**LONG.**	**LAT.**	**LONG.**	**LAT.**	**LONG.**
D10	30.11	257.57	0.87	-13.84	-0.04	-0.99	15.63	10.40

The 5th order remains the best results,

    Table 10: Residuals at the check points in region 5 from the different orders

	Residuals in cm							
	3 rd order		**4 th order**		**5 th order**		**6 th order**	
Pts	**LAT.**	**LONG.**	**LAT.**	**LONG.**	**LAT.**	**LONG.**	**LAT.**	**LONG.**
E1	10.15	123.02	-0.34	6.08	39.85	-59.02	1564.79	-872.06
E2	-2.21	277.22	0.02	-7.74	-35.96	51.67	-1654.46	859.97
E3	7.64	49.18	-0.46	3.12	20.26	-28.14	864.54	-425.56
E4	2.14	-135.55	0.25	7.97	-24.71	35.29	-1161.23	543.75
E5	16.20	-207.58	-0.32	-18.92	0.87	-0.59	-5.02	8.78
E6	0.71	-346.39	-0.06	-9.49	21.03	-29.48	925.17	-388.62
E7	-7.93	48.03	-0.01	-2.25	15.55	-21.24	612.65	-271.59
E8	4.88	69.43	0.50	-3.88	-18.11	24.68	-692.72	326.51
E9	-0.33	-66.15	0.14	-9.53	20.44	-28.76	695.09	-345.71
E10	15.30	-65.48	-0.11	-9.55	-38.00	53.85	-1671.79	895.29
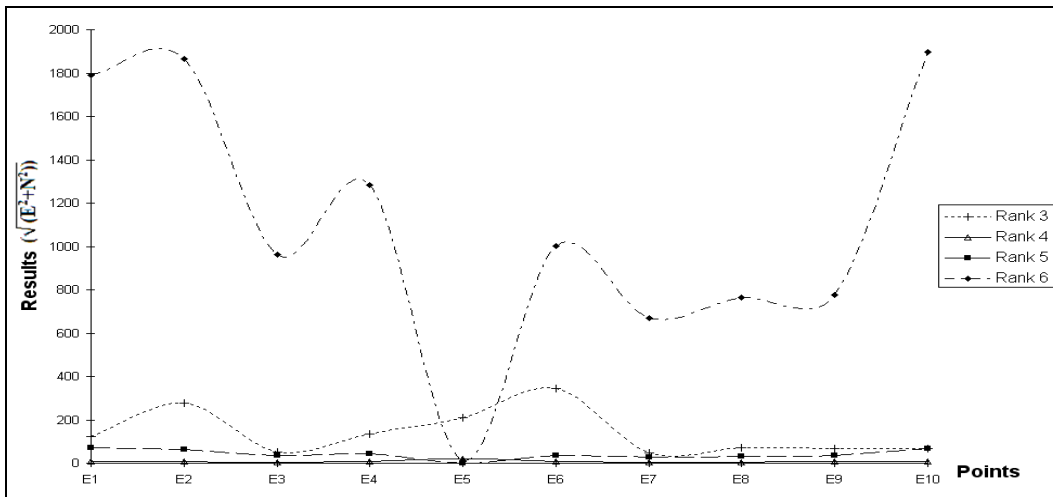
The 4th order polynomial gave the best results.
It is noticed that, within all cases "regions", the best results matched with the 4th and the 5th polynomial ranks where the outputs are illustrated at the preceding set.

	From $(\varphi, \lambda)_{WGS84}$ to UTM	From UTM to $(\varphi, \lambda)_{WGS84}$
Region 1	5th order	5th order
Region 2	5th order	5th order
Region 3	4th order	5th order
Region 4	4th order	5th order
Region 5	5th order	4th order

Also, the relatively peak values were corresponding to points "A6, B7, C6, C10, D3, E6, E10". This is due to the position of these points, where all of these points are near to the regions border. Despite the fact that the results are performing well, additional researches still need to be done in order to deal with this special behavior of border points.

Conclusions

After analyzing all the results over all the examined regions, it can be concluded that the obtainable data set with respect to the WGS84 datum can be converted to the UTM grids using the polynomial coefficients. The fifth order polynomial in the regions (1, 2, 5) and the fourth order polynomial in the regions (3,4) gave the best results with minor residual values that insignificantly affecting the precision of the coordinates.

The non familiar users of geodetic datums and map projection can easily use the developed software. In the case that $(\varphi, \lambda)_{WGS84}$ coordinates are available and their corresponding UTM values are needed, the file of $(\varphi, \lambda)_{WGS84}$ will be the input and the output will be the required file of UTM coordinates, or vice versa.

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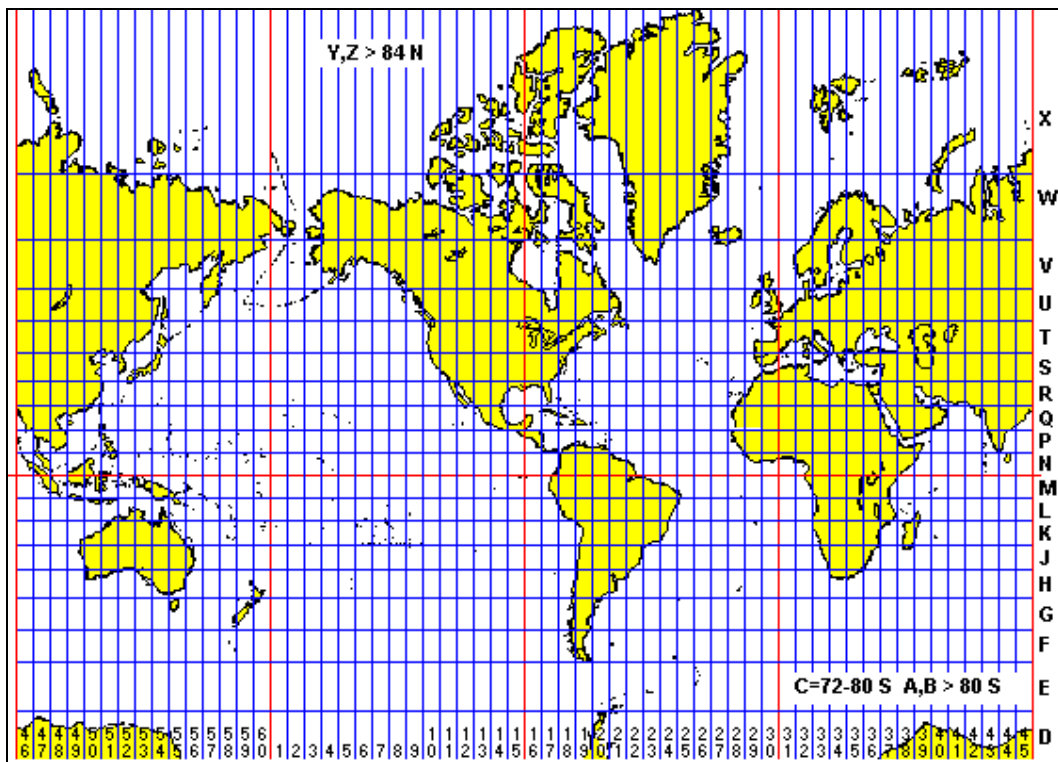
Figure (1): The zones of the Universal Transverse Mercator
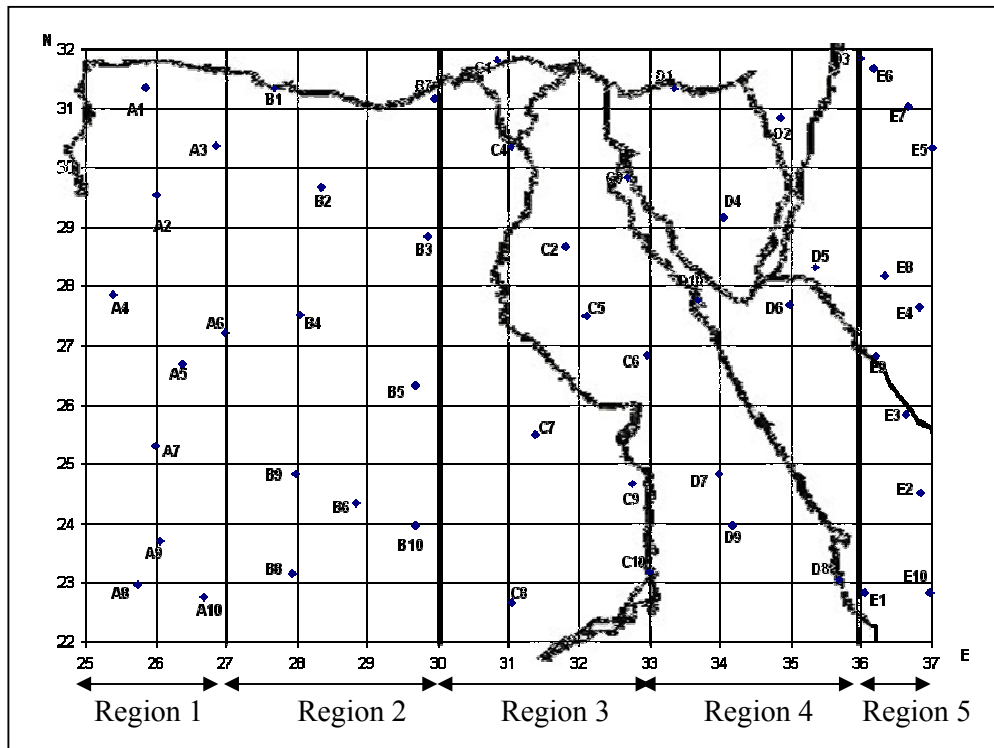

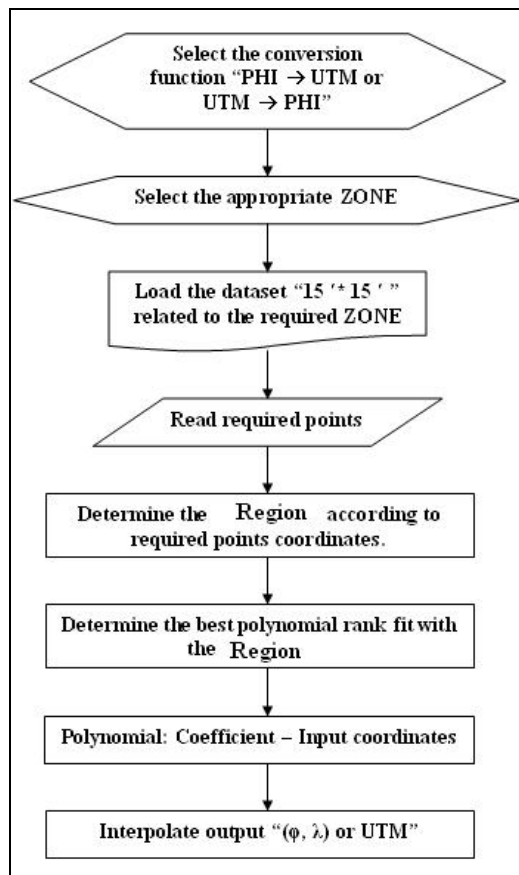**Figure (2): The proposed 5 regions for Egypt**

Fig (3) Flow chart represents the process of executing the required output